1. Given :

$V_{hr} = \frac{1}{20} \cdot V_{min}$.

$T_{hr} = 12$ hrs

$r_{min} = 10\text{cm} = 0.1\text{m}$ $T_{min} = 1$ hr.

To Find : $r_{hr} = ?$

Formula : $v = r\omega$

$V_{hr} = \frac{1}{20} V_{min}$.

$r_{hr} \cdot \omega_{hr} = \frac{1}{20} \cdot V_{min} \cdot \omega_{min}$

$V_{hr} \times \frac{2\pi}{12} = \frac{1}{20} \times \frac{1}{10} \times \frac{2\pi}{1}$

$r_{hr} = \frac{12}{200} = 6\text{cm}$.

Ans : Length of the hour hand of clock = 6cm.

2. Given :

$T_{seconds} = 60$ s.

$T_{min} = 60$ minutes $= 3600$ seconds

$T_{hr} = 12$ hrs $= 12 \times 3600$ s.

To Find : $\omega_s = ?$ $\omega_{min} = ?$ $\omega_{hr} = ?$

Formula : $\omega = 2\pi / T$.

(a) $\omega_s = \frac{2\pi}{T_s} = \frac{2\pi}{60} = \frac{2 \times 3.142}{60} = \frac{6.284}{60}$

$\omega_s = \frac{1.047}{10} = 0.105$ rad/s.

(b) $\omega_{min} = \frac{2\pi}{T_{min}} = \frac{2 \times 3.142}{3600} = \frac{3.142}{18} \times 10^{-2}$

$= \text{Al} (\log 3.142 - \log 18) \times 10^{-2}$

$= \text{Al} (0.4972 - 1.2553) \times 10^{-2}$

$= \text{Al} (\bar{1}.2419) \times 10^{-2}$

$= 1.746 \times 10^{-3}$

$\therefore \omega_{min} = 1.74 \times 10^{-3}$ rad/s.

(c) $\omega_{hr} = \frac{2\pi}{T_{hr}} = \frac{2 \times 3.142}{12 \times 3600} = \frac{3.142}{216} \times 10^{-2}$

$= \text{Al} (\log 3.142 - \log 216) \times 10^{-2}$

$= \text{Al} (0.4972 - 2.3345) \times 10^{-2}$

$= \text{Al} (\bar{2}.1627) \times 10^{-2}$

$= 1.454 \times 10^{-4}$

$= 14.5 \times 10^{-5}$ rad/s.

Ans : (a) $\omega_s = 0.105$ rad/s

(b) $\omega_{min} = 1.74 \times 10^{-3}$ rad/s

(c) $\omega_{hr} = 14.5 \times 10^{-5}$ rad/s.

3. Given :

$V_1 = 3V_2$ $\quad r_2 = r_1 - 8$

To Find : $d = ?$

Solution : $V = r\omega$.

$V_1 = r_1 \omega$

and $V_2 = r_2 \omega$.

From given data.

$r_2 = r_1 - 8$

$\therefore V_2 = (r_1 - 8)\omega$ ——— (1)

From given data

$V_1 = 3V_2$ ——— (2)

Substituting equation (1) in equation (2)

$r_1 \omega = 3(r_1 - 8)\omega$

$\therefore r_1 = 3r_1 - 24$

$\therefore -2r_1 = -24$

$\therefore r_1 = 12.$

$\therefore$ The diameter of the disc is $2r_1 = 24$ cm.

$d = 24$ cm.

Ans : The diameter of the disc = 24cm.

4. Given :

$d = 1$m $\quad \therefore r = 0.5$m

$f = 120$ rpm $= 2$ rps.

To Find : $V = ?$, $\omega = ?$

Solution :

(a) For a point on the rim :

$r = 0.5$m

$\omega = 2\pi f$

$= 2\pi \times 2$

$\omega = 4\pi$ rad/s.

$V = 2\omega$

$V = 0.5 \times 4 \times r$

$V = 6.28$ m/s.

(b) For a point on the rim :

~~$r = 0.5$ m~~ $r = 0.5$

~~$\omega = 2\pi f$~~

(b) For a point halfway to the centre.

$r = \frac{0.5}{2} = 0.25$ m.

$\omega = 4\pi$ rad/s.

$V = r\omega$

$V = 0.25 \times 4\pi$

$V = 3.14$ m/s.

5. Given :

$r = 0.25$ m. $f = 240$ rpm $= 4$ rps.

$f' = 330$ rpm

$= \frac{330}{6}$ rpm

$= 5.5$ rps.

To Find : $\omega = ?$, $V = ?$, $a = ?$ $\alpha = ?$

Solution :

(a) $\omega = 2\pi f.$

$= 2\pi \times 4.$

$\omega = 8\pi$ rad/s.

(b) $V = r\omega$

$= 0.25 \times 8\pi$

$V = 2\pi$ m/s.

(c) $\omega_2 = 2\pi f'$

$= 2\pi \times 5.5$

$= 11\pi$ rad/s.

$\therefore \alpha = \frac{\omega_2 - \omega_1}{t} = \frac{11\pi - 8\pi}{10}$

$= \frac{3\pi}{10} = 0.3\pi$ rad/s^2

(d) $a = r\alpha$

$= 0.25 \times 0.3\pi$

$= 0.075\pi$ m/s^2

6. Given :

$\alpha = 1$ rad/s^2, $\omega_1 = 5$ rad/s, $\omega_2 = 15$ rad/s.

To Find : $\theta = ?$

Solution :

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$, $\alpha = \frac{\omega_2 - \omega_1}{t}$

$\alpha = \frac{\omega_2 - \omega_1}{t}$

$t = \frac{\omega_2 - \omega_1}{\alpha} = \frac{15 - 5}{1} = 10$ s.

$\theta = 5 \times 10 + \frac{1}{2} \times 1 \times (10)^2$

$\theta = 100$ rad.

7. Given :

$r = 7000$ km $= 7 \times 10^6$ m.

$T = 2$ hrs $= 7200$ s.

$\omega = ?$ $\quad V = ?$ $\quad a_{cp} = ?$

Solution :

$$\omega = \frac{2\pi}{T} = \frac{2 \times 3.142}{7200}$$

$$= 8.73 \times 10^{-4} \text{ rad/s.}$$

$$V = r\omega = 7 \times 10^6 \times 8.73 \times 10^{-4}$$

$$= 61.11 \times 10^2$$

$$= 6.1 \times 10^3$$

$$= 6.1 \text{ km/s.}$$

$$a_{cp} = V\omega$$

$$= 6.1 \times 10^3 \times 8.73 \times 10^{-4}$$

$$= 5.32 \text{ m/s}^2.$$

8. Given :

$m = 2$ kg $\quad f = 300$ rpm $= 5$ rps.

$r = 1.5$ m.

To find : $V = ?$, $\quad a = ?$ $\quad F = ?$

Solution :

$$V = 2\pi rf$$

$$= 2 \times 3.142 \times \frac{15}{10} \times 5 \times 10^{-1}$$

$= 15 \times 31.42 \times 10^{-1}$

$= 471.30 \times 10^{-1}$

$V = 47.13$ m/s

$a = v^2/r$

$= (47.13)^2/15$

$= 2221.24/1.5$

$a = 1480.83$ m/s^2

$F = ma$

$= 2 \times 1480.83$

$= 2161.64$ N.

9. Given :

$m = 45$ kg, $F_{max} = mg$, $m = 0.1$ kg, $r = 5$m

$F_{max} = 45 \times 9.8$

$F_{max} = 441$ N.

To Find : $f = ?$

Solution :

$F = m\omega^2 r$

$= m(2\pi f)^2 r$

$441 = 0.1 \times (2 \times 3.142 \times f)^2 \times 5$

$441 = \frac{1}{10} \times 4 \times 9.87 \times f^2 \times 5$

$441 = 19.74 f^2$

$f^2 = \frac{441}{(3.142)^2 \times (\sqrt{2})^2}$

$\therefore f = \frac{21}{3.142 \times \sqrt{2}} = \frac{21}{4.4}$

$f = 4.77$ rps

Ans : Greatest number of revolutions which the string can make without breaking = 4.77 rps.

10. Given :

$m = 0.5\ kg = 5 \times 10^{-1}\ kg$

$r = 1.25\ m = 125 \times 10^{-2}\ m$

$T = 5$ sec.

To find : a) Tension = ?

b) If tension = 150 N, v = ? and T = ?

Solution :

$$\text{Tension} = mr\omega^2$$

$$= mr \times \frac{4\pi^2}{T^2}$$

$$= \frac{5 \times 10^{-1} \times 4 \times \pi^2 \times 125 \times 10^{-2}}{25}$$

$$= 25 \times 4\pi^2 \times 10^{-3} = \pi^2 \times 10^{-1} = 0.986\ N$$

$$\text{Tension} = \frac{mv^2}{r}$$

$$150 = \frac{5 \times 10^{-1} \times v^2}{125 \times 10^{-2}}$$

$$150 \times 25 \times 10^{-1} = v^2$$

$$v = \sqrt{15 \times 25}$$

$$= 19.36\ m/s$$

$$T = \frac{2\pi}{\omega} \qquad \left(v = r\omega,\ \omega = \frac{v}{r}\right)$$

$$= \frac{2\pi r}{v}$$

$= \frac{2\pi \times 12 \times 10^{-2}}{19.36}$

$= 40.54 \times 10^{-2}$

$T = \underline{0.4054 \text{ sec.}}$

11. Given :

$v = 27 \text{ km/h} = 27 \times \frac{5}{18} = \frac{15}{2} \text{ m/s.}$

$r = 10 \text{ m.}$

To find = $\theta = ?$

Solution :

$v = \sqrt{rg \tan\theta}$

$\tan\theta = \frac{v^2}{rg} = \frac{\left(\frac{15}{2}\right)^2}{10 \times 9.8}$

$= \frac{225}{4} \times \frac{10}{10 \times 98}$

$= \frac{225}{392}$

$= \underline{0.57397}$

$\tan\theta = 0.5740$

$\theta = \tan^{-1}(0.5740)$

$\underline{\theta = 29^\circ 51'}$

12. Given :

$v = 120 \text{ km/hr} = \frac{120 \times 5}{18.3} \text{ m/s} = \frac{100}{3} \text{ m/s.}$

$g = 9.8 \text{ m/s}^2$

$l = 3 \text{ km}$ $\quad \because 2\pi r = 3000 \Rightarrow r = \frac{3000}{2\pi} = \frac{1500}{\pi} \text{ m.}$

Solution :

$$\tan\theta = \frac{v^2}{rg}$$

$$= \left(\frac{100}{3}\right)^2$$

$$\neq \frac{98}{10} \times \frac{1500}{3.142}$$

$$= \frac{10}{9} \times \frac{31.42}{98 \times 1500}$$

$$= \frac{3142}{13230} = 0.2374.$$

$$\tan\theta = 13^\circ 21'$$

13. Given:

$r = 30\,m$, $\theta = 30^\circ$, $g = 9.8\ m/s^2$

To Find: $v_{max} = ?$

Solution:

$$v_{max} = \sqrt{rg\tan\theta}$$

$$= \sqrt{30 \times 9.8 \times \tan 30^\circ}$$

$$= \sqrt{3 \times 98 \times \frac{1}{\sqrt{3}}}$$

$$= \sqrt{98\sqrt{3}}$$

$$= \sqrt{98 \times 1.732}$$

$$= \sqrt{49 \times 2 \times 1.732}$$

$$= 7\sqrt{3.464}$$

$$= 7 \times 1.861$$

$= 13.027$

$V_{max} = 13.03$ m/s.

15. Given:

$r = 600$ m, $V = 180$ km/hr, $g = 9.8$ m/s^2

$V = 50$ m/s.

To find: $\theta = ?$

Solution: $\tan\theta = \frac{V^2}{rg}$

$= \frac{(50)^2}{600 \times 9.8}$

$= \frac{2500}{600 \times 9.8}$

$= \frac{125}{294}$

$= 0.4252$

$\theta = 23^\circ 2'$

16.

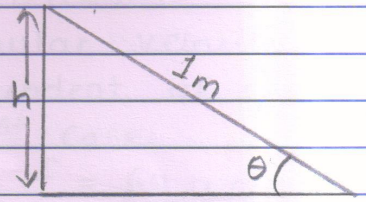
Date: _____

Name: _____

Std: _____ Roll No: _____ Subject: _____ Marks: _____

15. Given :

$r = 50\,m$, $v = 36\ km/h = 36 \times \frac{5}{18}\ m/s = 10\ m/s$.



$h = ?$

Solution : $v = \sqrt{rg \tan\theta}$

$$\tan\theta = \frac{v^2}{rg} = \frac{(10)^2}{50\,(9.8)}$$

$$= \frac{100}{50(9.8)}$$

$$\tan\theta = 0.2040$$

$$\theta = 11^\circ 3'$$

$$\sin\theta = \frac{h}{l}$$

$$h = \sin\theta$$

$$= \sin 11^\circ 31'$$

$$= 0.1997 \simeq 0.2\ m$$

Elivation = 0.2 m

16. Given :

$f = 180 \text{ rpm} = 3 \text{ rps}.$

$r = 2 \times 10^{-2} \text{ m}$

To find : $\mu = ?$

Solution :

$$\mu = \frac{v^2}{rg}$$

$$= \frac{(2\pi r f)^2}{rg}$$

$$= \frac{4\pi^2 f^2 r}{g}$$

$$= \frac{4 \times (3.142)^2 \times 9 \times 2 \times 10^{-2}}{9.8}$$

$$= \frac{4 \times 9.87 \times 9 \times 2 \times 10^{-2}}{9.8}$$

$$\mu \cong 0.72.$$

$$\mu = 0.72.$$

17. Given :

$r = 7 \times 10^{-2} \text{ m}$

$f = 60 \text{ rpm} = 1 \text{ rps}.$

To find : $f = ?$, When $r = 12 \text{ cm}$

Solution :

Since force acting on the coin will remain the same, irrespective of its distance from axis,

(i) $F_1 = F_2$

$m v_1 w_1 = m v_2 w_2$

$(2\pi r_1 f_1)(2\pi f_1) = (2\pi r_2 f_2)(2\pi f_2)$

$$4\pi^2 r_1 f_1^2 = 4\pi^2 r_2 f_2^2$$

$$f_2^2 = \frac{r_1 f_1^2}{r_2}$$

$$f_2^2 = \frac{7 \times (60)^2}{12}$$

$$f_2 = \underline{45.82 \text{ rpm}}$$

(ii) When another coin is kept on the coin, its mass increases.
But angular velocity (or frequency of rotation) is independent of mass, hence it is same as is in Ist case.

$\therefore f = \underline{60 \text{ r.p.m.}}$

13. Given :

$m = 2 \text{ kg}$ $\quad r = 0.9 \text{ m}$ $\quad V_{top} = 3 \text{ m/s}$.

Solution :

(i) $$T_{top} = \frac{mv^2}{r} + mg\cos\theta$$

$$= \frac{2 \times 9}{0.9} + (-2 \times 9.8) \quad [\cos 180^\circ = -1]$$

$$= 20 + (-19.6)$$

$$T_{top} = \underline{0.4 \text{ N}}$$

(ii) $V_{centre} = \sqrt{3rg}$

$$= \sqrt{3 \times 0.9 \times 9.8}$$

$$= \sqrt{3^2 \times 3 \times 2 \times 49 \times 10^{-2}}$$

$$= 21 \times 10^{-1} \times 2.45$$

$$V_{centre} = \underline{5.145 \text{ m/s}}$$

$T_{centre} = \frac{mv^2}{r} + mg\cos\theta.$

$= \frac{2\times(5.145)^2}{0.9} - 19.6 \times 0, \quad (\cos 90^\circ = 0).$

$= \frac{265\times 2}{9} + 0$

$= \underline{58.89\ N.}$

$V_{bottom} = \sqrt{5rg} = \sqrt{5\times 9.8\times 0.9}$

$= \sqrt{5\times 2\times 49\times 10^{-2}\times 3^2}$

$= 0.7\times 3\times 3.16.$

$V_{bottom} = 6.64\ m/s.$

$T_{bottom} = \frac{mv^2}{r} + mg\cos\theta.$

$= \frac{2\times(6.64)^2}{0.9} + 19.6\cos 0^\circ$

$= \frac{2\times 44.09}{0.9} + 19.6(1)$

$= \frac{88.18}{0.9} + 19.6.$

$= 97.98 + 19.6$

$T_{bottom} = \underline{117.58\ N.}$

19. Let v be velocity of car, on top

$$\frac{mv^2}{r} = mg$$

$$v^2 = rg \quad \text{——— (1)}$$

As the car runs down the incline, K.E. goes on increasing, while P.E. decreases.

$$\frac{1}{2}mv^2 = mgh$$

$$v^2 = 2gh \quad \text{——— (2)}$$

From (1) and (2)

$$rg = 2gh$$

$$2h = r.$$

$$h = \frac{r}{2} \text{ units.}$$

20. Given :

$m = 0.2\ kg$, $l = 0.5m$, $r = 25 \times 10^{-2}\ m$

To Find : $v = ?$, $T = ?$

Solution :

$$r = l \sin\theta$$

$$\sin\theta = \frac{r}{l} = \frac{25 \times 10^{-2}}{5 \times 10^{-1}} = 5 \times 10^{-1} = 0.5 = \frac{1}{2}$$

$\Rightarrow \theta = 30^\circ$

$\tan\theta = \tan 30^\circ = \frac{1}{\sqrt{3}}$ and $\cos\theta = \frac{\sqrt{3}}{2}$

$v = \sqrt{rg\tan\theta}$

$= \sqrt{25\times10^{-2}\times9.8\times\frac{1}{\sqrt{3}}}$

$= 0.5\sqrt{\frac{9.8}{1.732}}$

$= 0.5 \times 2.38$

$v = 1.19\ m/s = 119\ cm/s.$

$T = 2\pi\sqrt{\frac{l\cos\theta}{g}} = 2\times3.142\sqrt{\frac{0.5\times\sqrt{3}}{9.8\times2}}$

$= 7.284 \times \sqrt{\frac{0.866}{19.6}}$

$= 7.284 \times 0.21$

$\underline{T = 1.53.}$