ॐ विनायक क्लासेस
VINAYAK CLASSES
DEGREE & DIPLOMA ENGINEERING
XI-XII [Science / Commerce]
Date : ____

e : Gravitation.

: ____ Roll No. : ____ Subject : ____ Marks : ____

1. Data :

Mass of the two spheres :

$m_1 = 60\ kg$, $m_2 = 60\ kg$

Distance between their centres,

$r = 0.4\ m$. $F = 4.905 \times 10^{-7}\ N$

To Find : $G = ?$

Solution :

$$F = G \frac{m_1 m_2}{r^2}$$

$$\therefore G = \frac{F \times r^2}{m_1 \times m_2}$$

Or, $$G = \frac{4.905 \times 10^{-7} \times 0.16}{60 \times 20}$$

$$= 6.54 \times 10^{-11}\ Nm^2/Kg^2$$

2. Data:

Mass of the earth = M, Radius = R

Acceleration due to gravity on earth's surface = g.

Mass of the planet = M', Radius $R' = 2R/5$,

Acceleration due to gravity on planet's surface $g' = g/4$

$$\therefore \frac{R'}{R} = \frac{2}{5} \text{ and } \frac{g'}{g} = \frac{1}{4} \quad \ldots \text{(1)}$$

To Find: $M' = ?$

Solution:

In case of earth, $g = \frac{GM}{R^2}$ ... (2)

In case planet, $g' = \frac{GM'}{R'^2}$ ... (3)

Taking ratio of equation (2) and (3)

$$\frac{g'}{g} = \frac{M'}{M} \times \left(\frac{R}{R'}\right)^2$$

Or,

$$\frac{1}{4} = \frac{M'}{M} \times \left(\frac{5}{2}\right)^2$$

$$\therefore M' = \frac{M}{25}$$

3. Data :

Mass of the satellite, $m' = 400$ kg

Radius of the orbit $= 5R/2$

Radius of the earth $= R$

Gravitational force (F) on mass of 1 kg on the earth's surface = 10N.

Since $F = G\frac{Mm}{R^2}$,

$\therefore 10 = G\frac{M \times 1}{R^2}$ Or, $\frac{GM}{R^2} = 10$

To Find : F' (satellite) = ?

Solution :

$F'(\text{satellite}) = G\frac{Mm'}{(5R/2)^2}$

$= \frac{4}{25}\frac{GM}{R^2}m' = \frac{4}{25} \times 10 \times 400$

$= 640$ N.

4. Data :

On earth's surface :

$W = mg = 50\,N$ $g = 9.8\,m/s^2$,

radius of earth = R

To Find : $W' = ?$ $(h = R/4)$

Solution :

We have $g' = g\left(\frac{R}{R+h}\right)^2$

$= g\left(\frac{R}{R+(R/4)}\right)^2 = g \times \frac{16}{25}$

$\therefore W' = mg' = m\left(\frac{16}{25}g\right) = \frac{16}{25}mg$

$= \frac{16}{25} \times 50$

$= 32\,N.$

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5. Data :

Acceleration due to gravity = g

To Find : Percentage change in g if new radius of the earth

$R' = R - 0.02R = 0.98R.$

Solution :

We have $g = \frac{GM}{R^2}$

$\therefore g' = \frac{GM}{(0.98R)^2} = \frac{GM}{0.9604R^2}$

$= 1.04 \times \frac{GM}{R^2}$

$= 1.04g.$

$\therefore$ % change in g, $= \frac{g'-g}{g} \times 100$

$= \frac{1.04-1}{1} \times 100$

$= 4\%$

Value of g will increase by 4%.

6. Data :

Acceleration due to gravity = g.

At depth, $d = 300$ km, $g = g'$

$R = 6400$ km

To Find : $h = ?$ if $g = g'$

Solution :

At height h, $g' = g\left(1 - \frac{2h}{R}\right)$

At depth d, $g' = g\left(1 - \frac{d}{R}\right)$

$\therefore g\left(1 - \frac{2h}{R}\right) = g\left(1 - \frac{d}{R}\right)$

Or, $1 - \frac{2h}{R} = 1 - \frac{d}{R}$

Or, $h = \frac{d}{2} = \frac{300}{2}$

$= 150$ km.

7. Data :

$R = 6400\ km = 6.4 \times 10^6\ m$

Acceleration due to gravity $= g$

To Find : $d = ?$ Where $g' = 1\%$ of $g = \frac{g}{100}$

Solution :

$$g' = g\left(1 - \frac{d}{R}\right)$$

$$\frac{g}{100} = g\left(1 - \frac{d}{R}\right)$$

$$\frac{1}{100} = 1 - \frac{d}{R}$$

Or, $\frac{d}{R} = 1 - \frac{1}{100} = \frac{99}{100}$

Or, $d = \frac{99 \times R}{100} = \frac{99 \times 6400}{100}$

$= 6336\ km.$

8. Data :

Mass = 1 kg

Weight = 1 N.

R = 6400 km

To Find : Distance from the centre of the earth = ?

Solution :

At depth d, $g' = g\left(1 - \frac{d}{R}\right)$

Weight = mg'

$\therefore mg' = 1$ N.

Or, $mg\left(1 - \frac{d}{R}\right) = 1$ N.

Or, $1 \times 9.8\left(1 - \frac{d}{R}\right) = 1$

Or, $1 - \frac{d}{R} = \frac{1}{9.8} = 0.102$

Or, $\frac{d}{R} = 0.898$

Or, $d = 6400 \times 0.898 = 5747.2$ km

Or, distance from the centre of the earth

$= 6400 - 5747.2 = 652.8$ km.

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9. Data :

$T = 24$ hrs, $R = 6400$ km

To Find : $g_P - g_E = ?$

Solution :

Value of g : $g_\lambda = g - \omega^2 R\cos^2\lambda$

λ = latitude. At equator $\lambda = 0$, at pole $\lambda = 90°$

For earth :

$\omega = \frac{2\pi}{24 \times 60 \times 60}$

$= 7.3 \times 10^{-5}$ rad/s

At pole $\lambda = 90°$, $\cos\lambda = 0$

$\therefore g_P = g = 9.8\ m/s^2$

At equator $\lambda = 0$, $\cos\lambda = 1$.

$g_E = g - (6.4 \times 10^6 \times 53.3 \times 10^{-10})$

$g_P - g_E = 0.0341\ m/s^2$

10. Data :

$T = 24$ hrs, $R = 6400$ km

To Find : Angular velocity of the earth $\omega = ?$

Solution :

At the equator bodies to be weightless

Centripetal force = mg.

Or, $mR\omega^2 = mg$.

Or, $\omega = \sqrt{\frac{g}{R}} = \sqrt{\frac{9.8}{6.4 \times 10^6}}$

$= 1.237 \times 10^{-3}$ rad/s.

Period, $T = \frac{2\pi}{\omega} = \frac{2 \times 3.14}{1.237 \times 10^{-3}}$

$= 5076.8$ sec $= 1.4$ hrs.

11. Data :

$R = 6400\ km = 6.4 \times 10^6 m$

$\rho = 5500\ kg/m^3$

$G = 6.67 \times 10^{-11}\ Nm^2/kg^2$

To Find : $g = ?$

Solution :

Mass of the earth,

M = volume x mean density

$M = \frac{4\pi R^3}{3} \times \rho$ [$\because$ Volume of a sphere $= 4\pi R^3/3$]

$= \frac{4 \times 3.14 \times (6.4 \times 10^6)^3 \times 5500}{3}$

$= 6.036 \times 10^{24}$

We have, $g = \frac{GM}{R^2}$

$= \frac{6.67 \times 10^{-11} \times 6.036 \times 10^{24}}{(6.4 \times 10^6)^2}$

$= 9.83\ m/s^2$

12. Data :

Acceleration due to gravity = g.

R = 6400 km

To Find : h = ? ($g' = 0.04g$)

Solution :

We have : $g' = g\left(\frac{R}{R+h}\right)^2$

$$\therefore 0.04g = g \times \left(\frac{6400}{6400+h}\right)^2$$

Taking square root $0.2 = \frac{6400}{6400+h}$

Or, $6400 + h = \frac{6400}{0.2} = 32000$

Or,

$h = 32000 - 6400$

$= 25,600$ km.

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13. Data :

Period $T = 27.3$ days.

$= 27.3 \times 24 \times 60 \times 60$

$= 2.36 \times 10^6$ secs.

$g = 9.8 \ m/s^2$

$R = 6400 \ km = 6.4 \times 10^6 m$

To Find : $r = ?$

Solution :

We have, $T^2 = \frac{4\pi^2}{gR^2} \times r^3$

$\therefore r^3 = \frac{gR^2T^2}{4\pi^2}$ Or, $r = \left(\frac{gR^2T^2}{4\pi^2}\right)^{1/3}$

$$r = \left(\frac{9.8 \times (6.4 \times 10^6)^2 \times (2.36 \times 10^6)^2}{4 \times (3.14)^2}\right)^{1/3}$$

Or, $r = 3.84 \times 10^5$ km.

14. Data :

$v = 34.9$ km/s $= 34.9 \times 10^3$ m/s.

$M_s = 1.97 \times 10^{30}$ kg.

Mass of the planet = m.

$G = 6.67 \times 10^{-11}$ Nm²/kg²

To find : T = ?

Solution :

Since the planet moves in a circular orbit around the Sun the centripetal force experienced by it is equal to the force of gravitational attraction between the planet and the sun.

so $\frac{mv^2}{r} = G\frac{M_s m}{r^2}$

or, $r = \frac{GM_s}{v^2}$

Orbital speed $v = \frac{2\pi r}{T}$

so $T = \frac{2\pi r}{v} = \frac{2\pi GM_s}{v^3}$

$= \frac{2 \times 3.14 \times 6.67 \times 10^{-11} \times 1.97 \times 10^{30}}{(34.9 \times 10^3)^3}$

$= 1.94 \times 10^7$ sec.

$= \frac{1.94 \times 10^7}{60 \times 60 \times 24}$

$= 225$ days.

15. Data :

Let the distance of the earth from sun be $r_E = r$,
then distance of pluto from sun, $r_p = 40 \times r$.
Period of earth $T_E = 1$ year

To Find : $T_p = ?$

Solution :

We have : $T^2 = \frac{4\pi^2}{gR^2} \times r^3$

$\therefore T_E^2 = \frac{4\pi^2}{g_s R_s^2} \times r_E^3$

and $T_p^2 = \frac{4\pi^2}{g_s R_s^2} \times r_p^3$.

Taking ratio of the above equations.

$\frac{T_p^2}{T_E^2} = \frac{r_p^3}{r_E^3}$ Or, $\frac{(T_p)^2}{(1\ year)^2} = \frac{(40r)^3}{r^3}$

Or, $(T_p)^2 = (40)^3 = 64000$

Taking square root,

$T_p = \sqrt{64000} = 253$ years.

16. Data :

$R = 6400\ km = 6.4 \times 10^6 m$

$g = 9.8\ m/s^2$

$G = 6.67 \times 10^{-11}\ Nm^2/kg^2$

To find : $\rho = ?$

Solution :

We have : $g = \frac{GM}{R^2}$

Or, $g = \frac{G \times \left(\frac{4}{3}\pi R^3 \times \rho\right)}{R^2}$

Or,

$g = \frac{4\pi G R \rho}{3}$ Or, $\rho = \frac{3g}{4\pi G R}$.

$\rho = \frac{3 \times 9.8}{4 \times 3.14 \times 6.67 \times 10^{-11} \times 6.4 \times 10^6}$

$= 5483\ kg/m^3$

17. Data :

$r = 3.84 \times 10^5$ km $= 3.84 \times 10^8$ m.

$M = 5.96 \times 10^{24}$ kg

$m = 7.30 \times 10^{22}$ kg

$F = 1.968 \times 10^{20}$ N.

To Find : $G = ?$

Solution :

We have : $F = G \frac{Mm}{r^2}$

$\therefore G = \frac{F \times r^2}{Mm}$

$$G = \frac{1.968 \times 10^{20} \times (3.84 \times 10^8)^2}{5.96 \times 10^{24} \times 7.3 \times 10^{22}}$$

$= 6.67 \times 10^{-11}$ Nm²/kg².

18. Data :

(Satellite) A, $r_1 = r$

(Satellite) B, $r_2 = 4r$

To find : $(v_c)_A : (v_c)_B = ?$

Solution :

We have : $v = \sqrt{\frac{gR^2}{r}}$

$(v_c)_A = \sqrt{\frac{gR^2}{r_1}} = \sqrt{\frac{gR^2}{r}}$

$(v_c)_B = \sqrt{\frac{gR^2}{r_2}} = \sqrt{\frac{gR^2}{4r}}$

$\frac{(v_c)_A}{(v_c)_B} = \sqrt{\frac{gR^2}{r}} \Big/ \sqrt{\frac{gR^2}{4r}}$

$= \sqrt{\frac{4}{1}}$

$= \frac{2}{1}$

19. Data :

$G = 6.67 \times 10^{-11}\ Nm^2/kg^2$

$\rho = 5500\ kg/m^3$.

$R = 6400\ km = 6.4 \times 10^6 m$

To find : orbital speed, $v = ?$

Solution :

We have $v = \sqrt{\frac{GM}{R+h}}$

Since the satellite is close to earth,

$h = 0$ $\therefore v = \sqrt{\frac{GM}{R}}$

Mass of the earth $= \frac{4}{3}\pi R^3 \times \rho$

$\therefore v = \sqrt{\frac{G \times 4 \times \pi \times R^3 \times \rho}{3R}}$

$= 2R \times \sqrt{\frac{\pi \times \rho \times G}{3}}$

$= 2 \times 6.4 \times 10^6 \times \sqrt{\frac{3.14 \times 5500 \times 6.67 \times 10^{-11}}{3}}$

20. Data :

$R = 6400 \text{ km} = 6.4 \times 10^6 m.$

$h = 800 \text{ km} = 0.8 \times 10^6 m.$

$r = R + h = 6.4 \times 10^6 + 0.8 \times 10^6$

$= 7.2 \times 10^6 m/s. \quad g = 9.8 m/s^2$

To Find : (i) $v = ?$ (ii) $T = ?$

Solution :

(i) $v = \sqrt{\frac{gR^2}{R+h}} = \sqrt{\frac{g}{R+h}} \times R$

Or, $v = \sqrt{\frac{9.8}{7.2 \times 10^6}} \times 6.4 \times 10^6$

$= 7.47 \times 10^3 m/s$

$= 7.47 \text{ km/s}.$

(ii) $T = \frac{2\pi r}{v}$

$= \frac{2 \times 3.14 \times 7.2 \times 10^6}{7.47 \times 10^3}$

$= 6053$ secs.